Future Destiny of Quintessential Universe and Constraint on Model from Deceleration Parameter

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Abstract:

The evolution of the quintessence in various stages of the universe, the radiation-, matter-, and quintessence-dominated, is closely related with the tracking behavior and the deceleration parameter of the universe. We gave the explicit relation between the equation-of-state of the quintessence in the epoch of the matter-quintessence equality and the inverse power index of the quintessence potential, obtained the constraint on this potential parameter come from the present deceleration parameter, i.e., a low inverse power index. We point out that the low inverse power-law potential with a single term can not work for the tracking solution. In order to have both of the tracker and the suitable deceleration parameter it is necessary to introduce at least two terms in the quintessence potential. We give the future evolution of the quintessential universe.

1 Introduction

The important observation on the high redshift Type Ia supernovas in 1998 is to discover that the universe is expanding accelerated with the negative deceleration parameter[1,2]

$$q_0 \equiv -\ddot{R}/(RH^2) = -0.33 \pm 0.17. \tag{1}$$

The universe must contain some new dark energy which equation-of-state $w = p/\rho$ is negative. In spite of the small non-zero cosmology constant ($w_\lambda = -1$) is one of the possible explanations, people does not satisfy this scheme due to its bad coincidence and fine-turning problems. A popular candidate is the quintessence[3] ($-1 < w_q < 0$), which is a slowly varied scalar field ϕ and has an inverse power-law potential $V = m^{4+\beta}\phi^{-\beta}$, where we call the parameter β as the inverse power index of the quintessence. We see that its minimum, i.e., vacuum has zero energy, this tallies with such a thought that the true cosmological constant should be zero due to some unknown profound reason. Zlatev et al. find out that the quintessence has

a nice property, the tracking behavior, and its potential initial values with almost 100 different orders can converge to a common evolving final state, i.e., the tracking solution[4]. This scheme can well solve the coincidence problem. The idea of the quintessence received enough attentions, many papers did further researches on the quintessence[5,6], and developed many new ideas[7,8]. The inverse power index β is an important parameter for quintessence, the quintessence will not be able to come into the tracking situation up to the present time if β is too small, for example $\beta < 5$ as shown by Ref.[4]. An important problem is whether the present experiment, more concretely from the cosmic deceleration parameter, have added some restriction on this inverse power index. In this paper we shall research this interesting problem in detail. We shall find out the equations-of-states of the quintessence in various evolving stages of the universe at first, then express the deceleration parameter in these equation-of-states. The observation result of the deceleration parameter will constrain the inverse power index of the quintessence up to $\beta \leq 2$. We shall give the evolution of the quintessence energy density, the cosmic scale factor and the equation-of-state of the quintessence in the future.

2 Equation-of-states of quintessence

The evolution of the quintessence in the radiation dominated or the matter dominated epochs is dependence on its equation-of-states $w_q = (\beta w_B - 2)/(\beta + 2)$, where w_B is the equation-of-state of the background. We cite this formula directly, which has obtained in Ref.[4]. The best tracking behavior requires that the evolution velocity of the quintessence in the radiation dominated epoch ($w_r = 1/3$) is the equal to (or larger than) the one of the matter ($w_m = 0$), i.e., $w_q^{(r)} = (\beta/3 - 2)/(\beta + 2) = 0$, then we obtain $\beta = 6$. The equation-of-state of the quintessence in the matter dominated epoch is $w_q^{(m)} = -(\beta/2 + 1)^{-1}$, which evolution is slower than one of the matter.

As the universe expanses, the quintessential potential energy will overweigh one of the matter and become quintessence dominated, it is important to know what is the equation-of-state of the quintessence in the quintessence dominated epoch, which can be obtained by analyzing the equation of field motion $3H\dot{\phi} = \beta m^{4+\beta}\phi^{-\beta-1}$ and $3M_p^2H^2 = m^{4+\beta}\phi^{-\beta}$, where $M_p \equiv (8\pi G)^{-1/2} = 2.4 \times 10^{18}$GeV is the Planck energy. Note that in this stage the field rolls slowly and the potential energy is dominated, these allow us to take the above approximation. We suppose that the solution is a simple power-law type of the time, $\phi \propto t^\alpha$. Comparing the exponential about time t in the two sides of the equations, we get $\alpha = 2/(\beta + 4)$. This power value confirms further the conditions of the slow rolling $\ddot{\phi} \ll 3H\dot{\phi}$ and potential dominated $\dot{\phi}^2/2 \ll V$, and the approximation is reasonable. This result shows that the quintessence field value is increasing slowly as the age of the universe increases. Then we obtain that the evolution of the quintessence energy density ρ_q as the

cosmic scale factor R is

$$\rho_q = \rho_{q0}\{1 + \frac{6\tau}{\beta+4}\ln\frac{R}{R_0}\}^{-\beta/2}, \tag{2}$$

and the cosmic scale factor expands as the universe time

$$R = R_0 \exp\{\frac{\beta+4}{6\tau}[(\frac{t}{t_0})^{4/(\beta+4)} - 1]\}, \tag{3}$$

where the constant $\tau^{-1} = \frac{3}{2}H_0 t_0 \simeq 1$ due to the matter dominated[9], where H_0 is the Hubble constant in the time of the beginning of the quintessence dominated and t_0 is the universe age in the same time. It is obviously that the decreasing of the quintessence energy density as the increasing of the cosmic scale factor is very slow, its limited behavior is similar with the cosmological constant. Eqs.(2-3) determined the future destiny of our universe. We can obtain the equation-of-state of the quintessence as a function in the cosmic scale factor

$$w_q(R) \equiv -1 - \frac{d\ln\rho}{3d\ln R} = -[(4 + 6\ln\frac{R}{R_0})^{-1}\beta + 1]^{-1}. \tag{4}$$

The equation-of-state of the quintessence in the beginning of the quintessence dominated is obtained as $w_q^{(q)} = w_q(R_0) = -(\beta/4+1)^{-1}$.

The Type Ia Supernovas measured by Perlmutter et al. have average redshift $z \simeq 0.4$, in that epoch the universe undergoes the phase transition from the matter dominated to the quintessence dominated. Watching on the variation from the equation-of-state of the quintessence in the matter dominated epoch $w_q^{(m)}$ to one in the quintessence dominated epoch $w_q^{(q)}$ carefully, we see that in the middle time, in which the matter density is nearly equal to the quintessence density, the equation-of-state of the quintessence should be $w_q^{(e)} = -(\beta/\gamma+1)^{-1}$, here we take $\gamma = 3$ may be a reasonable approximation. It is this equation-of-state of the quintessence $w_q^{(e)}$ that should be applied in the formula of the deceleration parameter of the recent observation of the high redshift supernovas.

3 Deceleration parameter

Now we see the deceleration parameter

$$q_0 = \frac{1}{2}\sum\Omega_i + \frac{3}{2}\sum w_i\Omega_i, \tag{5}$$

where $\Omega_i \equiv \rho_i/\rho_c$ are the ratios of various component densities to the critical density of the universe. We tested the correctness of this formula, even if in the case of the existence of the quintessence. We cite this correct formula directly, which appeared in Ref.[10]. Considering the red shift effect, we have

$$q_0 = \frac{1}{2}\Omega_{m0}\cdot(1+z)^3 + (\frac{1}{2} - \frac{3}{2}(\beta/\gamma+1)^{-1})\Omega_{q0}\cdot(1+z)^{3-3/(\beta/\gamma+1)}. \tag{6}$$

Noted the curvature term Ω_u with $w_u = -1/3$ has just be canceled. When $\beta = 0$ and $z = 0.4$ we obtain $q_0 \simeq \frac{4}{3}\Omega_{m0} - \Omega_{\lambda 0}$, and Ref.[1] gives the observation result $0.6q_0 = 0.8\Omega_{m0} - 0.6\Omega_{\lambda 0} = -0.20 \pm 0.10$. In the later we shall use an equivalent result eq.(1). Using the eqs.(6) and (1) one can discuss what values the density ratios should take in the figure of Ω_{m0} and $\Omega_{\lambda 0}$, like the treatment in Ref.[1].

However we can think that as an approximation we can take $\Omega_{m0} = 0.3$, $\Omega_{q0} = 0.7$ at first, then use q_0 formula to constrain the parameter β. Thus we obtain an important restriction, i.e., parameter β is less than 2. Here we used the assumption of the flat universe predicted by the inflation models[11]. Let us look at some concrete data. For example, if $\beta = 1$, then $w_q^{(e)} = -0.75$ and $q_0 = -0.15$ which is out off the lower limit of the observation value; if $\beta = 1/2$, then $w_q^{(e)} = -0.86$ and $q_0 = -0.22$ in according to eq.(6). If we take $\Omega_{m0} = 0.2$, $\Omega_{q0} = 0.8$, $z = 0.4$, $\beta = 2$, then $w_q^{(e)} = -0.60$ and $q_0 = -0.20$; or we take $\beta = 3$ and the same other parameters, then $w_q^{(e)} = -0.50$ and $q_0 = -0.06$. Anyhow the inverse power index must be less than 3. In fact it seems impossible that the sum of the densities of the cold dark matter and the baryon matter is so small like $\Omega_{m0} = 0.2$ in according to the estimated matter quantity from the X-ray observation on the cluster of the galaxies[12], $\Omega_{m0} = 0.35 \pm 0.07$.

If the inverse power index of the quintessence is $\beta = 2$, the equation-of-state of the quintessence in the radiation dominated era is $w_q^{(r)} = -1/3$, and the quintessence energy density is $\rho_q \propto a^{-2}$, it decreases too slowly. At the early stage of the universe, due to $\rho_q/\rho_\gamma = (1+z)^{-2}$ and very high red-shift, require that the quintessence energy density must be very low. In this case it is very easy for the quintessence to produce overshot behavior and can not begin to track even in very late time, therefore the initial condition can not be adjusted in wide rang, and then it can not solve the coincidences problem. In fact the ref.[4] has given a conclusion that the inverse power index must be larger than 5. Thus the quintessence with the low inverse power index lost its attracting property.

During the derivation, we use some reasonable approximations, we think the further exact results will not affect our main conclusion.

4 Two term potential

Of course we can use more complicated potential to overcome this difficulty, for example $V = m_6^{10}\phi^{-6} + m_2^6\phi^{-2}$. When field ϕ is small, the first term is dominated, it has a good tracking behavior and wide adjusting rang of the initial condition. When field ϕ becomes large, the second term is dominated, the quintessence has the suitable the equation-of-state w_q for the deceleration parameter of the supernovas. However, since the second term has a lower inverse power index, the energy scale parameter m_2 has to be rather small, this may induce the fine-turning problem. On the other hand, in this scheme, we must turn finely the relative ratio of the mass parameters m_6 and m_2, let it undergo a transition from the high inverse power term

dominated to the low inverse power term dominated just before the time of the matter-quintessence equality.

Let us consider at what redshift this transition should happen. The tracking solution should satisfy the equation[4]

$$V'' = \frac{9}{2}\frac{\beta+1}{\beta}(1-w_q^2)H^2, \tag{7}$$

then we obtain the relation between the quintessential field value ϕ and the quintessential energy faction Ω_q in the matter-dominated epoch

$$\phi^2 = \frac{2}{3}\beta(\beta+2)^2(\beta+4)^{-1}\Omega_{q0}(1+z)^{-2}M_p^2, \tag{8}$$

When $\beta = 6$, then $\phi_6 \simeq 5\Omega_{q0}^{1/2}(1+z)^{-1}M_p$ and when $\beta = 2$, then $\phi_2 \simeq 2\Omega_{q0}^{1/2}M_p$, the transition should happen in the time of $\phi_6 < 0.6\phi_2$. Therefore we obtain the redshift $z > 3$. The more early this transition from high to low inverse power terms process, the more narrow the adjusting rang of the initial condition is. Why does the nature arrange these two transitions in such order?

The order magnitude of the mass parameters are $m_6 \simeq 10^5$GeV and $m_2 \simeq$ 10MeV, if the low inverse power term is $m_1^5\phi^{-1}$, then $m_1 \simeq$ 1keV. As a comparison, the cosmological constant $\Lambda = m_0^4$ has $m_0 \simeq 10^{-3}$eV. We see that the fine turning problem is relaxed[13].

5 Conclusion

The important problem is whether the observation data can constrain some models. We obtain the various equation-of-states of the quintessence in the different stages of the universe, specially the equation-of-states of the quintessence $w_q^{(e)} = -(\beta/3+1)^{-1}$ in the time of the matter-quintessence equality. Using it we give the restriction on the inverse power index $\beta \leq 2$ of the quintessence potential from the deceleration parameter of the high redshift supernovas.

The two functions of the tracking and the present quintessential energy have to be achieved by the separated different two terms in the quintessence potential respectively. Why the God takes so refinement arrangement?

It is hopeful that the deceleration parameter which will be more accurate in future, combining other observation, specially the total density ratio $\Omega_0 = \Omega_m + \Omega_q$ from the position of the first Doppler peak of CMBR[14,15], will give the more strict constrain on the potential parameter of the quintessence.

If somebody does not approve of the non-terseness of the two term potential of the quintessence, one can explore the exponential potentials or other more complicated ones. To search new ideas to replace one of the cosmological constant is the interesting challenge problem.

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References:

[1] S.J.Perlmutter et al., Nature 391(1998)51.
S.J.Perlmutter, et al., astro-ph/9812133.
[2] A.G.Riess, et al., Astron.J.116(1998)1009.
[3] B.Ratra and P.L.E.Peebles, Phys.Rev.D37(1988)3406.
[4] I.Zlatev, L.Wang and P.J.Steinhardt, Phys.Rev.Lett.896 (1999);
P.J.Steinhardt, L.Wang, and I.Zlatev, Phys.Rev.D59(1999)123504.
[5] M.S.Turner and M.White, Phys.Rev.D56(1997)4439.
[6] A.R.Liddle and R.J.Scherrer, Phys.Rev.D59(1999)23509.
[7] R.R.Caldwell, R.Dave and P.J.Steinhardt,
Phys.Rev.Lett.80(1998)1582.
[8] P.J.E.Peebles and A.Vilenkin, astro-ph/9810509.
[9] E.W. Kolb and M.S. Turner,
The Early Universe, Addison Wesley, 1990.
[10] M.S.Turner, astro-ph/9904049; astro-ph/9912211.
[11] A.Linde, *Particle Physics and Inflationary Cosmology*,
1990 by Harwood Academic Publishers.
[12] J. Mohr et al, Astrophys.J., in press (1999) (astro-ph/9901281).
[13] S.Weinberg, Rev.Mod.Phys.61(1989)61.
[14] M.Kamionkowski and A.Kosowsky, astro-ph/9904108.
[15] S. Dodelson and L. Knox, astro-ph/9909454.